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By considering the area preserving geometric transformations in the configuration space of electrons moving in the lowest Landau level (LLL) we arrive at the Chern–Simons type Lagrangian. Imposing the LLL condition, we get a scheme with the complex gauge fields and transformations. Quantum theory for the matter field in LLL is considered and formal expressions for Read's operator and Laughlin wave function are presented in the second quantized form.

I. INTRODUCTION

The lowest Landau level comprises the minimal energy states of the charged particle moving in the $x-y$ plane in the presence of an orthogonal magnetic field $\mathbf{B} = \mathrm{rot}\,\mathbf{A}$. Corresponding wave functions satisfy equation

$$-iD_{\bar{z}}(\mathbf{A})\psi_{\mathrm{L}}(\mathbf{r}) \equiv [(\hat{p}_x + i\hat{p}_y) - e(A_x + iA_y)]\psi_{\mathrm{L}}(\mathbf{r}) = 0 \tag{1}$$

Electrons in the LLL form a special kind of incompressible quantum fluid exhibiting such an interesting phenomenon as the fractional quantum Hall effect (FQHE) (see *e.g.* Ref.[1]). A physically attractive way to understand the formation of corresponding quantum states is the picture of composite quasiparticles *i.e.* fermions (bosons) carrying odd (even) number of elementary magnetic flux quanta [2]. The field-theory realization of this idea leads to the model, where the matter is interacting with the Chern–Simons (CS) gauge field [3,4].

It must be noted however that in the most part of presentations corresponding field theories include higher Landau levels and the subsequent projection on the LLL is needed [5]. As an interesting alternative one can consider the formulation including solely LLL states and fields.

Below we propose a scheme, where the Chern–Simons vector field emerges as a result of some geometric transformations in the complexified configuration space of the electron in the LLL. These transformations are restricted by two conditions: they must be area preserving and the electron wave function must obey the LLL condition (1). The third assumption implies that the one-particle charge density distribution does not change under the about transformation.

The resulted field theory corresponds to the earlier proposed schemes [6,7] with a Chern–Simons field in the holomorphic gauge and the field theory realizations of Read's operator [8] in terms of the non-unitary singular transformation.

In the present paper we reformulate the problem in terms of the fermion fields obeying the LLL condition (1). We propose the specific Lagrangian which in the certain limit describes the matter field in the LLL. As an output we construct the LLL analogue of Read's operator and the Laughlin wave function [9] for FQHE.

Notations:. In the $x-y$ plane together with the Cartesian coordinates $x^i = x_i$ we use the complex ones: $z = x + iy$, $\bar{z} = x - iy$ with a metric tensor $g_{z\bar{z}} = \frac{1}{2}$. Electrons have a charge $e = -1$, mass m and move in the area Ω in the homogeneous magnetic field pointing down the $\hat{\mathbf{z}}$ axis: $\partial_x A_y - \partial_y A_x = -B < 0$.

II. GEOMETRIC TRANSFORMATIONS

The classical Lagrangian for the planar electron moving in the magnetic field is given by

$$L = \frac{m}{2}\dot{x}^i\dot{x}^i - \dot{x}^i \cdot A_i(x). \tag{2}$$

The corresponding canonical Hamiltonian reads

$$H_c = \frac{1}{2m}\pi_i\pi_i \equiv \frac{1}{2m}\left(\pi_z\pi_{\bar{z}} + \pi_{\bar{z}}\pi_z\right),$$

where $\pi_i = p_i + A_i$ is a kinetic momentum.

In quantum mechanics the wave function of the lowest energy state satisfies the equation:

$$\hat{\pi}_{\bar{z}}\psi_{\mathrm{L}}(\mathbf{r}) \equiv \frac{1}{2}(\hat{\pi}_1 + i\hat{\pi}_2)\psi_{\mathrm{L}}(\mathbf{r}) = 0. \tag{3}$$

The solutions to this equation depend on the gauge field $\mathbf{A}(\mathbf{r})$. For example, in the symmetric gauge

$$A_1 = \frac{B}{2}y, \qquad A_2 = -\frac{B}{2}x, \tag{4}$$

which is well adapted to the use of complex variables, the Eq. (3) looks as follows:

$$\left(\frac{\partial}{\partial \bar{z}} + \frac{B}{4}z\right)\psi_{\mathrm{L}}(\mathbf{r}) = 0.$$

The standard LLL wave function is of the form

$$\psi_{\mathrm{L}}(z, \bar{z}) = e^{-(B/4)|z|^2} f(z), \tag{5}$$

with $f(z)$ a function of z only.

The Eq. (3) can be viewed as a condition imposed on the physical states by the constraint dynamics (in the Dirac's [10] sense), and at the classical level these constraints mean the vanishing of the kinetic momentum:

$$\pi_i \approx 0. \tag{6}$$

Remark, that one can deduce these constraints starting with the singular Lagrangian [11]

$$\mathcal{L} = -\dot{x}^i A_i(x), \tag{7}$$

which is a zero mass limit of (2).

Since the corresponding canonical Hamiltonian vanishes, the quantum dynamics is completely governed by constraints (6). At the same time one cannot impose operator constraints

$$\hat{\pi}_i|\Psi_{\mathrm{L}}\rangle = 0 \qquad \langle\Psi_{\mathrm{L}}|\hat{\pi}_i = 0,$$

because they do not commute among themselves

$$0 = \langle\Psi_{\mathrm{L}}|[\hat{\pi}_1, \hat{\pi}_2]_-|\Psi_{\mathrm{L}}\rangle = iB \neq 0.$$

Instead, constraints can vanish only "weakly", *i.e.*

$$\hat{\pi}_{\bar{z}}|\Psi_{\mathrm{L}}\rangle = 0, \qquad \langle\Psi_{\mathrm{L}}|\hat{\pi}_z = 0.$$

This is consistent with classical Eqs. (6), as well as the corresponding quantum averages vanish

$$\langle\Psi_{\mathrm{L}}|\hat{\pi}_i|\Psi_{\mathrm{L}}\rangle = 0.$$

Consider the zero mass (*i.e.* LLL) Lagrangian

$$\mathcal{L} = -\dot{x}^1(\alpha x^2) - \dot{x}^2(-\beta x^1) \tag{8}$$

corresponding to the gauge potentials

$$A_1 = \alpha x^2, \qquad A_2 = -\beta x^1; \qquad \alpha + \beta = B.$$

Perform the coordinate transformation

$$x^i \rightarrow \nu_{(i)}(x^i + \epsilon^{ik} f_k(\mathbf{r})), \tag{9}$$

where $\mathbf{f}(\mathbf{r})$ is a vector field, and the nonzero $\nu_{(i)}$'s are scaling factors (there is no summation over the index (i)).

Under this transformation the area is changed according to the standard rule

$$\Omega \rightarrow \Omega_\nu = \nu\Omega + \nu\int_\Omega \mathrm{d}\mathbf{r}\epsilon^{ik}\partial_i f_k(\mathbf{r}) + \nu\int_\Omega \mathrm{d}\mathbf{r}[\partial_1 f_1 \cdot \partial_2 f_2 - \partial_1 f_2 \cdot \partial_2 f_1] \tag{10}$$

where $\nu = \nu_{(1)} \cdot \nu_{(2)}$

The Lagrangian (8) is also transformed

$$\mathcal{L} \to \mathcal{L}'_\nu = -\dot{x}^i(A'_i + a_i) + \frac{\vartheta}{2}\epsilon^{ik}a_i\dot{a}_k. \tag{11}$$

In the last expression the vector-potential

$$A'_1 = \nu\alpha x^2, \qquad A'_2 = -\nu\beta x^1, \qquad \mathrm{rot}\,\mathbf{A}'(\mathbf{r}) = \nu B,$$

and we have introduced the rescaled vector field $a_i(\mathbf{r}) = -\nu B f_i(\mathbf{r})$ and the constant $\vartheta = (\nu B)^{-1}$.

In the Lagrangian (11) the field $\mathbf{a}(\mathbf{r}, t)$ is treated as a dynamical variable, hence

$$\dot{\mathbf{a}} = \frac{\mathrm{d}}{\mathrm{dt}}\mathbf{a}(\mathbf{r}, t).$$

Now we would like to interpret the transformations (9), (10) and (11) in the light of FQHE characteristics and identify ν with the FQHE filling factor

$$\nu = \frac{2\pi N}{B\Omega} = \frac{1}{2p+1}.$$

Suppose first that the area is not changed, *i.e.* $\Omega \to \Omega_\nu = \Omega$. This hypothesis is motivated by the incompressibility of the quantum Hall fluid. Consequently

$$(\nu - 1)\Omega = \frac{1}{B}\int \mathrm{d}\mathbf{r}\epsilon^{ik}\partial_i a_k(\mathbf{r}) \tag{12}$$

(here and below we consider transformations (9) with a vanishing third term on the r.h.s. in Eq. (10)).

Secondly, consider a particle at $\mathbf{r}$ in the presence other particles at $\mathbf{r}_I$ ($I = 1, 2, ..., N$), which at the moment, are treated as punctures in the plane. Associate to the each puncture a singularity, and suppose that the deformation field $a_i(\mathbf{r})$ satisfies the equation

$$\epsilon^{ik}\partial_i a_k(\mathbf{r}) = \omega\sum_{I=1}^{N}\delta^{(2)}(\mathbf{r} - \mathbf{r}_I). \tag{13}$$

Eqs. (12) and (13) give

$$(\nu - 1)\Omega = \frac{\omega}{B}N$$

and consequently

$$\omega = 2\pi\frac{\nu - 1}{\nu} = -4\pi p.$$

Remark, that the magnetic field $B_1 = \nu B$ corresponds to the IQHE with the filling factor $\nu_1 = 1$ and suppose, that the wave function corresponding to the Lagrangian (11) satisfies the LLL condition

$$\pi'_{\bar{z}}|\psi'_\nu\rangle = (p_{\bar{z}} + A'_{\bar{z}})|\psi'_\nu\rangle = 0 \tag{14}$$

This assumption is equivalent to the following condition imposed on the field a_i:

$$a_{\bar{z}} = 0. \tag{15}$$

In the symmetric gauge ($A_z = \frac{i}{4}\bar{z}$, $A_{\bar{z}} = -\frac{i}{4}z$) one easily finds that the sought after vector field $\mathbf{a}(\mathbf{r})$ can be presented as a pure gauge

$$a_z = \partial_z\Lambda, \qquad a_{\bar{z}} = \partial_{\bar{z}}\Lambda,$$

where

$$\Lambda(z|z_I) = 2ip\sum_I \ln(z - z_I). \tag{16}$$

Indeed, such a vector field guarantees vanishing third term in Eq. (10), has a singular curl (13) and corresponds to Eq. (15).

It must be emphasized, that the field $a_i(\mathbf{r})$ defined as above, can not be real. It means that the transformation (9) requires the complexification of coordinates. Hence we treat z and $\bar{z}$ as independent variables spanning a complex two-dimensional plane [12,13] (in the physical subspace $\bar{z} = z^\star$).

With the help of the singular gauge transformation

$$a_i(\mathbf{r}) \to a_i(\mathbf{r}) - \partial_i \Lambda(\mathbf{r}|\mathbf{r}_I) = 0 \tag{17}$$

the vector field $\mathbf{a}$ can be gauged out. In parallel Lagrangian (11) will be also transformed

$$\mathcal{L}'_\nu \to \mathcal{L}_1 = -\dot{x}^i A'_i(\mathbf{r}). \tag{18}$$

The resulted Lagrangian (18) corresponds to the LLL electron in the magnetic field B_1.

In quantum mechanics transformations (17) and (18) are accompanied by the wave function transformation

$$\psi'_\nu(\mathbf{r}) \to e^{i\Lambda(\mathbf{r}|\mathbf{r}_I)}\psi'_\nu(\mathbf{r}) = \psi_1(\mathbf{r}).$$

Hence

$$\psi'_\nu(\mathbf{r}) = \prod_I (z - z_I)^{2p}\psi_1(\mathbf{r}) \equiv R(\mathbf{r}|\mathbf{r}_1, .., \mathbf{r}_N)\psi_1(\mathbf{r}). \tag{19}$$

In the last expression the wave function $\psi'_\nu(\mathbf{r})$ corresponds to the Lagrangian (11), $R(\mathbf{r}|\mathbf{r}_1, .., \mathbf{r}_N)$ represents the first quantized form of Read's operator and $\psi_1(\mathbf{r})$ is the LLL wave function of the fermion in the magnetic field B_1

$$\psi_1(z, \bar{z}) = e^{-(B_1/4/)z\bar{z}} f(z). \tag{20}$$

In the space of physical states obeying the LLL condition (14) operator $\hat{\pi}'_z$ is given by the expression

$$\hat{\pi}'_z = \hat{p}_z + A'_z + 2ip \sum_I \frac{1}{z - z_I}, \tag{21}$$

and with the usual inner product, $\hat{\pi}'_{\bar{z}} = \hat{p}_{\bar{z}} + A'_{\bar{z}}$ and $\hat{\pi}'_z$ turn out not to be an Hermitean conjugate pair of operators.

Now following Ref.[14] modify the definition of the inner product, introducing

$$\langle\langle \Psi|\Psi'\rangle = \int \mathrm{d}^2 z \mu(z, \bar{z})\psi^\star(z, \bar{z})\psi'(z, \bar{z}),$$

where

$$\mu(z, \bar{z}) = \prod_I |z - z_I|^{-4p}. \tag{22}$$

With this inner product $\hat{\pi}'_{\bar{z}}$ and $\hat{\pi}'_z$ become Hermitean conjugate to each other

$$\langle\langle \Psi|\hat{\pi}'_{\bar{z}}|\Psi'\rangle = \langle\langle \Psi'|\hat{\pi}'_z|\Psi\rangle^\star.$$

Introduction of the measure (22) can be justified in the another way. Let us consider the theories corresponding to the Lagrangians (8) and (11) as equivalent ones. Since under the transformation (9) the area is preserved, it seems natural to suggest that the corresponding charge density distribution also remains unchanged. Using Eqs. (5), (19) and (20), for the charge density distribution one gets the expression

$$\varrho(\mathbf{r}) = \psi^\star_{\mathrm{L}}(\mathbf{r})\psi_{\mathrm{L}}(\mathbf{r}) = \psi^\ddagger_\nu(\mathbf{r})\psi_\nu(\mathbf{r}),$$

where

$$\psi_\nu(\mathbf{r}) = e^{-(B/4)z\bar{z}(1-\nu)} \prod_I (z - z_I)^{2p}\psi_1(\mathbf{r}), \tag{23}$$

and

$$\psi_\nu^\ddagger(\mathbf{r}) = \mu(z,\bar{z})\psi_\nu^\star(\mathbf{r}) = \prod_I |z - z_I|^{-4p}\psi_\nu^\star(\mathbf{r}).$$

With these prescriptions quantum-mechanical averages are defined by the integral (up to the normalization factor)

$$\langle\hat{\mathcal{O}}\rangle = \langle\langle\Psi_\nu|\hat{\mathcal{O}}|\Psi_\nu\rangle \equiv \int \mathrm{d}^2 z \psi_\nu^\ddagger(z,\bar{z})\hat{\mathcal{O}}\psi_\nu(z,\bar{z}) \tag{24}$$

and that constraints (6) vanish in the weak sense

$$\langle\langle\Psi_\nu|\hat{\pi}_i|\Psi_\nu\rangle = 0.$$

Now one can ask the question: what is the field theory counterpart of the above construction? From the Lagrangian (11) it follows the Euler–Lagrange equation for the field a_i:

$$\dot{a}_k = -\frac{1}{\theta}\epsilon_{kl}\dot{x}^l. \tag{25}$$

Substituting this in (11) we arrive at the effective Lagrangian

$$\mathcal{L}_{eff} = -\dot{x}^i(A'_i + \frac{1}{2}a_i), \tag{26}$$

where the curl of $a_i(\mathbf{r})$ satisfies (13).

It is not difficult to guess, that the corresponding field theory is given by the CS gauge Lagrangian:

$$\mathcal{L}_{CS} = -j^\mu(x)(A'_\mu(x) + a_\mu(x)) + \frac{\kappa}{2}\varepsilon^{\mu\nu\lambda}a_\mu(x)\partial_\nu a_\lambda(x) \tag{27}$$

(the Greek indices run over values 0, 1, 2).

This equivalence can be demonstrated easily, plugging the field equations

$$\kappa\varepsilon^{\mu\nu\lambda}\partial_\nu a_\lambda(x) = j^\mu(x)$$

back into (27). In the temporal gauge $a_0 = 0$ the corresponding effective Lagrangian

$$\mathcal{L}_{eff} = -j^\mu(x)(A'_\mu(x) + \frac{1}{2}a_\mu(x)) \tag{28}$$

is an exact analogue of (26). The Gauss law

$$\epsilon^{ik}\partial_i a_k(x) = \frac{1}{\kappa}j^0(x) \tag{29}$$

is identical with (13) if one puts:

$$\kappa = -\frac{1}{4\pi p} = -\frac{N}{2p\Omega}\theta. \tag{30}$$

So one can reconstruct the gauge Lagrangian with the Chern–Simons constant, determined by the requirement of the area conservation under the coordinate transformations (9).

III. READ'S OPERATOR AND FIELD THEORY IN LLL

In this item we would like to obtain the second quantized counterpart of the Eq. (19). The problem is that the space of LLL states is not complete and there is no direct way to write down the particle density operator with δ-type singularities corresponding to point-like excitations.

Denote by $\hat{\psi}_\nu(\mathbf{r})$ and $\hat{\psi}_1(\mathbf{r})$ the quantum fields corresponding to the wave functions under consideration. The matter fields in the lowest Landau level can be expanded in the series

$$\hat{\psi}_1(\mathbf{r}) = \sum_n^\infty \hat{f}_n u_n(\mathbf{r}) \qquad\qquad \hat{\psi}_1^\dagger(\mathbf{r}) = \sum_n^\infty \hat{f}_n^+ u_n^\star(\mathbf{r}),$$

where the system of orthonormal solutions $u_n(\mathbf{r})$ in the symmetric gauge is given by

$$u_n(\mathbf{r}) = \frac{1}{\sqrt{2\pi}\ell}\frac{1}{\sqrt{2^n n!}}\left(\frac{z}{\ell_1}\right)^n e^{-(1/4)|z/\ell_1|^2}, \qquad \ell_1^2 = \frac{1}{B_1},$$

and Fermi amplitudes satisfy standard relations

$$\hat{f}_n\hat{f}_m^+ + \hat{f}_m^+\hat{f}_n = \delta_{nm}.$$

The anti-commutator for Fermi fields in the LLL is expressed in terms of the bilocal kernel

$$[\hat{\psi}_1(\mathbf{r}'), \hat{\psi}_1^\dagger(\mathbf{r}'')]_+ = \sum_n^\infty u_n(\mathbf{r}')u_n^\star(\mathbf{r}'') = K_1(\mathbf{r}'|\mathbf{r}'') \tag{31}$$

instead of the customary $\delta(\mathbf{r}' - \mathbf{r}'')$ function. Hence the conventional charge density $\hat{\varrho}(\mathbf{r}) = \hat{\psi}_1^\dagger(\mathbf{r})\hat{\psi}_1(\mathbf{r})$ is not a good candidate for the sought after density operator.

Being the projection onto the space of LLL states, the operator $K_1(\mathbf{r}'|\mathbf{r}'')$ is not invertible. One can bypass this obstacle introducing the modified kernel

$$K_\xi(\mathbf{r}'|\mathbf{r}'') = \xi K_1(\mathbf{r}'|\mathbf{r}'') + (1-\xi)[\delta(\mathbf{r}'|\mathbf{r}'') - K_1(\mathbf{r}'|\mathbf{r}'')], \tag{32}$$

which coincides with (31) in the limit $\xi \to 1$. For $\xi \neq 1$ exists an inverse operator:

$$K_\xi^{-1}(\mathbf{r}'|\mathbf{r}'') = \frac{1}{\xi}K_1(\mathbf{r}'|\mathbf{r}'') + \frac{1}{1-\xi}[\delta(\mathbf{r}'|\mathbf{r}'') - K_1(\mathbf{r}'|\mathbf{r}'')]. \tag{33}$$

Now consider the action

$$\mathcal{A}_\xi = i\int \mathrm{d}t\mathrm{d}\mathbf{r}'\mathrm{d}\mathbf{r}''\psi_\xi^\star(t,\mathbf{r}')K_\xi^{-1}(\mathbf{r}'|\mathbf{r}'')\partial_t\psi_\xi(t,\mathbf{r}'') - \int \mathrm{d}t\mathrm{d}\mathbf{r}\psi_\xi^\star(t,\mathbf{r})V(\mathbf{r})\psi_\xi(t,\mathbf{r}) \tag{34}$$

where $V(\mathbf{r})$ is some one-particle potential.

Varying this action with respect to $\psi_\xi^\star(t,\mathbf{r})$ one gets the ξ-dependent equation

$$i\int \mathrm{d}\mathbf{r}'K_\xi^{-1}(\mathbf{r}|\mathbf{r}')\partial_t\psi_\xi(t,\mathbf{r}') = V(\mathbf{r})\psi_\xi(t,\mathbf{r}), \qquad \xi \neq 1. \tag{35}$$

The last equation can be inverted

$$i\partial_t\psi_\xi(t,\mathbf{r}) = \int \mathrm{d}\mathbf{r}'K_\xi(\mathbf{r}|\mathbf{r}')V(\mathbf{r}')\psi_\xi(t,\mathbf{r}'). \tag{36}$$

Thus we arrive at the Euler–Lagrange equation of motion for the field $\psi_\xi(t,\mathbf{r})$ corresponding to the action (34).

In the last expression one can readily go to the limit $\xi \to 1$ and obtain the single-particle Schrödinger equation for the matter field in the lowest Landau level

$$i\partial_t\psi_1(t,\mathbf{r}) = \int \mathrm{d}^2\mathbf{r}'K_1(\mathbf{r},\mathbf{r}')V(\mathbf{r}')\psi_1(t,\mathbf{r}'). \tag{37}$$

This equation first was derived by J. Martinez and M. Stone [15] using the variational formalism for the constrained system with the Lagrangian

$$L = \int \mathrm{d}^2\mathbf{r}\psi_1^\star(t,\mathbf{r})(i\partial_t - V(\mathbf{r}))\psi_1(t,\mathbf{r})$$

$$+\int \mathrm{d}^2\mathbf{r}([\psi_1(t,\mathbf{r}) - \int \mathrm{d}^2\mathbf{r}'K_1(\mathbf{r},\mathbf{r}')\psi_1(t,\mathbf{r}')\eta(t,\mathbf{r}')] + h.c).$$

Here $\eta(\mathbf{r})$ is the Lagrange multiplier introduced in order to enforce the LLL condition

$$\psi_1(t,\mathbf{r}) = \int d\mathbf{r}' K_1(\mathbf{r}|\mathbf{r}')\psi_1(t,\mathbf{r}') \tag{38}$$

(the last relation is equivalent to $D_{\bar{z}}(\mathbf{A}')\psi_1(\mathbf{r}) = 0$).

The canonical momentum is given by the nonlocal expression

$$\pi_\xi(t,\mathbf{r}) = \frac{\delta L}{\delta(\partial_t \psi_\xi(t,\mathbf{r}))} = i\int d\mathbf{r}' \psi^*_\xi(t,\mathbf{r}')K^{-1}_\xi(\mathbf{r}'|\mathbf{r}).$$

From the equal time anti-commutators

$$[\hat{\pi}_\xi(t,\mathbf{r}), \hat{\psi}_\xi(t,\mathbf{r}')]_+ = i\delta(\mathbf{r}-\mathbf{r}'), \qquad [\hat{\psi}_\xi(\mathbf{r}), \hat{\psi}^\dagger_\xi(\mathbf{r}')]_+ = M_\xi(\mathbf{r}|\mathbf{r}')$$

one sees, that the operator

$$\hat{R}_\xi(t,\mathbf{r}) = -i\hat{\pi}_\xi(t,\mathbf{r})\hat{\psi}_\xi(t,\mathbf{r}) \equiv \int d\mathbf{r}' \hat{\psi}^\dagger_\xi(t,\mathbf{r}')K^{-1}_\xi(\mathbf{r}'|\mathbf{r})\hat{\psi}_\xi(t,\mathbf{r}) \tag{39}$$

has a δ-like commutator with the matter field:

$$[\hat{R}_\xi(t,\mathbf{r}), \hat{\psi}_\xi(t,\mathbf{r}')]_- = -\delta(\mathbf{r}-\mathbf{r}')\hat{\psi}_\xi(t,\mathbf{r}). \tag{40}$$

Now one can suggest the limiting procedure, implying that $\lim_{\xi\to 1} \hat{\psi}_\xi(\mathbf{r}) = \hat{\psi}_1(\mathbf{r})$ is a LLL operator. Below we will operate with the needed quantities and relations for $\xi \neq 1$, and go to the limit $\xi \to 1$ only in the final expressions. Note that it is a usual procedure performed with the gauge fixing term in gauge theories. From the functional integral point of view the main contribution comes from the configurations satisfying the LLL condition (38), as well as $\frac{1}{1-\xi}$ terms are rapidly oscillating.

For $\xi \neq 1$ one can define the field operator

$$\hat{\psi}_{\nu,\xi}(\mathbf{r}) = e^{\Phi_\xi(\mathbf{r})}\hat{\psi}_\xi(\mathbf{r})$$

where

$$\Phi_\xi(\mathbf{r}) = -2p\frac{B_1}{4}|z|^2 + 2p\int d\mathbf{r}' \ln(z-z')\hat{R}_\xi(\mathbf{r}'),$$

and fields are taken at the fixed time (say $t=0$).

Define the "vacuum" state $\langle 0_\xi|\hat{\psi}^\dagger_\xi(\mathbf{r}) = 0$ and consider the bra-vector:

$$\langle 0_\xi|\hat{\psi}_\xi(\mathbf{r}_1)\cdots\hat{\psi}_\xi(\mathbf{r}_N)\cdot\hat{\psi}_{\nu,\xi}(\mathbf{r}) =$$

$$= \langle 0_\xi|e^{2p\int d\mathbf{r}' \ln(z-z')\hat{R}_\xi(\mathbf{r}')}e^{i\Lambda(z|z_I)}\hat{\psi}_\xi(\mathbf{r}_1)\cdots\hat{\psi}_\xi(\mathbf{r}_N)\cdot\hat{\psi}_\xi(\mathbf{r}) =$$

$$= e^{i\Lambda(z|z_I)}\langle 0_\xi|\hat{\psi}_\xi(\mathbf{r}_1)\cdots\hat{\psi}_\xi(\mathbf{r}_N)\cdot\hat{\psi}_\xi(\mathbf{r}). \tag{41}$$

In the last expression one can safely go to the limit $\xi = 1$ recovering the second quantized analogue of the Eq. (19).

At the same way one can consider the state:

$$\langle 0_\xi|\hat{\psi}_{\nu,\xi}(\mathbf{r}_1)\cdots\hat{\psi}_{\nu,\xi}(\mathbf{r}_N) = e^{-2p(B_1/4)\sum|z_I|^2}\prod_{I<K}(z_I - z_K)^{2p}\langle 0_\xi|\hat{\psi}_\xi(\mathbf{r}_1)\cdots\hat{\psi}_\xi(\mathbf{r}_N).$$

Introduce the ground state corresponding to fermions in the magnetic field B_1, totally filling up lowest Landau level

$$|GS\rangle = \prod_{j=0}^{N-1} f^+_j|0\rangle, \qquad N = \frac{B_1\Omega}{2\pi}.$$

Then

$$\lim_{\xi\to 1}\langle 0_\xi|\hat{\psi}_\xi(\mathbf{r}_1)\cdots\hat{\psi}_\xi(\mathbf{r}_N)|GS\rangle = e^{-(B_1/4)\sum|z_I|^2}\prod_{I<K}(z_I - z_K) \tag{42}$$

is the corresponding $\nu = 1$ wave function (Slater determinant).

Consequently, the limiting value of the matrix element

$$\lim_{\xi\to 1}\langle 0_\xi|\hat{\psi}_{\nu,\xi}(\mathbf{r}_1)\cdots\hat{\psi}_{\nu,\xi}(\mathbf{r}_N)|GS\rangle = e^{-(B/4)\sum|z_I|^2}\prod_{I<K}(z_I - z_K)^{2p+1} \tag{43}$$

reproduces the Laughlin wave function [9] for the fractional states with $\nu = \frac{1}{2p+1}$.

IV. CONCLUDING REMARKS

The aim of the present paper is two-fold. First, we tried to reveal the link between area preserving diffeomorphisms in FQHE [16,17] and CS gauge fields. Demanding that these transformations does not violate the quantum-mechanical LLL condition we arrive to the conclusion, that the CS field corresponds to the non-unitary similarity transformation between the integral and fractional quantum Hall systems [6,7].

Secondly, we attempted to find a field-theory form for the singular gauge transformations in terms of purely LLL field operators. Although we failed to give a direct solution to this task, an extended Lagrangian formalism and the limiting procedure is described, permitting to reproduce the needed second quantized quantities. Remark that the proposed scheme agrees with the earlier one [15] in the general aspects.

Concluding, we would like to note, that the field-theory constructions in the LLL (like Read's operator, Jains composite particles) can be naturally incorporated in the CS gauge theory with the complex gauge transformation group. In the present paper we discuss a version of this scheme, where the complex gauge transformations are induced by area preserving coordinate transformations in the lowest Landau level. It seems interesting to consider the same framework for bilayer quantum Hall systems and point particles with non-Abelian charges.

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